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Measurement and Effects of Vibration

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Preface

Following its successful conferences in East Anglia in 1985 and Bristol in 1988, SECED (The Society for Earthquake and Civil Engineering Dynamics) decided to hold the third in this series of international technical conferences at UMIST, Manchester, on 18–20 September 1991.

This book contains the 47 papers which were accepted for formal presentation at the conference. The choice of papers was made on the basis of over 80 abstracts offered, some of which were selected for presentation at a poster session. The papers are organised into nine groups as follows:

Session 1: Ground Motion and Field Studies (Papers 1–6)
Session 2: Analysis and Case Histories (Papers 7–11)
Session 3: Isolation and Damping (Papers 12–15)
Session 4: Blast (Papers 16–20)
Session 5: Members and Joints (Papers 21–25)
Session 6: Instrumentation and Testing (Papers 26–32)
Session 7: Impact (Papers 33–37)
Session 8: Piling and Demolition (Papers 38–43)
Session 9: Code Issues and Feedback (Papers 44–47)

This book contains a wealth of information and we hope that engineers and researchers will find it useful.

J. R. Maguire
Chairman,
SECED Conference Organising Committee

A COMPUTATIONAL STRATEGY FOR THE CRACKING PROCESS IN CONCRETE STRUCTURES UNDER SHOCK AND EARTHQUAKE LOADING CONDITIONS

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ABSTRACT

A computational strategy to the cracking process is presented for the finite element analysis of planar and axisymmetric reinforced concrete structures under transient and impulsive loading conditions. A smeared crack formulation is proposed which is based on the fixed crack approach. The crack initiation is controlled by a new rate dependent strain criterion. The postcracking regime is governed by non-linear reversible formulations to model concrete tension softening as a function of concrete fracture energy, tensile strength, cracking strain and crack characteristic length. Crack shear is allowed to take place using a non-linear model dependent on the strain across the crack. Practical recommendations are made to some of the problems associated with the smearing approach. A new crack monitoring algorithm, as part of a comprehensive computer program FEABRS, is developed which follows the cracking behaviour with the time stepping scheme. The solution of some dynamic problems, with beam and circular slab elements, are presented and a comparison is made with results from other sources.

INTRODUCTION

The low probability of incidence of transient and impulsive loads requires the design of structural elements such that some level of irreversible structural deformation and material damage is acceptable, provided that the overall structural integrity is maintained. For the non-linear finite element analysis of dynamically loaded concrete structures, constitutive relationships are required which take into account the inelastic response in compression, the degradation of stiffness due to cracking, the yielding of reinforcing steel, and the effects of high strain rate on the behaviour of concrete and steel. Such history and rate dependent relationships have been developed in [1] and have been presented with examples in [2, 3]. As the tensile cracking process plays a dominant role in the non-linear behaviour of concrete, the main attention in the present paper is paid to the description of a proposed computational strategy for the crack initiation and propagation process in reinforced concrete structures subjected to severe dynamic loads such as blast, impact and earthquakes.

CONCRETE TENSILE BEHAVIOUR AND NUMERICAL REPRESENTATION OF CRACKS

Concrete tensile strain-softening behaviour

The main characteristic of plain concrete behaviour is its low tensile strength. As a result, tensile cracking is considered as the major factor contributing to the non-linear behaviour of reinforced concrete structures. Several experiments [4, 5] have revealed that this response is primarily controlled by the formation of micro-cracks. Initially a limited number of such cracks develop anywhere in the specimen, but if the tensile stress or strain reaches a certain limit, all deformation due to micro-cracks will localize within a fracture zone. The stress gradually decreases while the strain increases. This phenomenon is known as a tensile strain-softening. The stress strain curve in tension is linear almost up to the peak stress followed by considerable softening. When the end of the descending strain softening branch is approached, the micro-cracks coalesce into one continuous macro-crack. From the study of different testing results carried out at higher strain rates than the conventional static tests, it was concluded [1] that increasing the stress or strain rate in tension or compression results in higher concrete strength. However, the tensile response of concrete is more strain rate sensitive than its compressive strength. The inequality of rate effects in tension and compression induces anisotropic effect. It has also been observed [4, 5] that with the increase of strain rate, the stress strain curves in tension are almost linear and show only curvature in the final phase of pre-cracking range before the activation of tension softening regime.

To account for this observed behaviour, concrete is modelled in tension here as a rate sensitive linear elastic strain softening material. That is, concrete behaves elastically until cracks initiate according to rate dependent cracking criterion and then the crack propagation process is governed by a fracture energy based softening rule in which tensile strength normal to the crack is gradually released in uniformity with the crack width.

Finite element representation of cracks

In the finite element context, two main approaches have been used for crack representation [6]: the discrete crack model and the smeared crack model. The choice between one or the other depends on the purpose of the finite element analysis, with regard to local or overall behaviour of a structure. The discrete concept fits our material conception of fracture since cracking is identified as a geometrical discontinuity. Conversely, it has been stated that a smeared representation might be realistic considering the bands of micro-cracks that blunt fracture in matrix-aggregate composites like concrete. The width of such bands, which occur at the tip of visible crack, has even been claimed to be a material property [7]. At present, however, it is difficult to judge these arguments since experimental detection of crack tip related micro-mechanical process in matrix-aggregate composites is scarce and the question remains whether these processes occur in a discrete manner or not.

Following the most common approach, a smeared crack model has been adopted here where cracked concrete is assumed to remain a continuum and the material properties are modified to account for the damage due to cracking. This approach has been chosen for the following reasons:

1. For severe dynamic loading conditions where the cracking patterns are difficult to predict, and also for situations where the scale of the representative continuum is large compared to the crack spacing, the smeared crack concept provides a realistic approach for distributed fracture representation compared with the discrete model which seems adequate for simple problems involving a few dominant cracks.
2. The proposed constitutive model is suitable for the description of concrete behaviour at the engineering level but not at the microscopic level which will necessitate the discrete approach.
3. The simplicity and the computational advantages of the smeared crack approach such as automatic generation of cracks without pre-defining or re-defining the finite element mesh, and also complete generality in allowing crack initiation in any direction.

CRACK INITIATION PROCESS

Strain rate sensitive cracking criterion
Simple criteria are commonly employed by most analysts to predict tensile fracture. The maximum tensile stress or strain theories [6] are frequently adopted for this purpose. Under severe dynamic conditions, the main limitation of the existing crack initiation criteria is that the strain rate effects are not included in spite of reported experimental results. For the present analysis, the tensile crack initiation is based on a rate sensitive criterion to distinguish the elastic behaviour from tensile fracture. The influence of deformation velocity on the material behaviour is taken into consideration by raising the cracking strain in relation to static condition using a rate sensitivity function as follows

$$\varepsilon_{td} = \phi_t \left(\dot{\varepsilon}_{eff}\right) \varepsilon_{ts} \tag{1}$$

in which ε_{td} is the dynamic cracking strain, ε_{ts} is the static cracking strain and $\phi_t(\dot{\varepsilon}_{eff})$ is the strain rate sensitivity function of concrete cracking and which is assumed to depend upon the effective strain rate. The least squares curve fitting was used to obtain a mathematical relation which fits the experimental results, found in the literature, for concrete tensile strength. Due to lack of sufficient experimental results, the effect of strain rate on the tensile strength and the cracking strain is assumed to be the same [4, 5]. The strain rate sensitivity function ϕ_t is found [1] to be

$$\phi_t\left(\dot{\varepsilon}_{eff}\right) = 2.23 + 0.404 \log \dot{\varepsilon}_{eff} + 0.0351 \left(\log \dot{\varepsilon}_{eff}\right)^2 \tag{2}$$

The advantage of this proposed crack initiation criterion over the other existing criteria is that concrete brittleness observed in dynamic tests, strain rate dependency and the inclusion of strain rate induced anisotropy, can all be considered.

Introduction of cracking process for plane and axisymmetric problems
For a previously uncracked point in a plane structure, cracks are assumed

to form in planes perpendicular to the direction of maximum tensile strain as soon as this strain reaches the specified concrete dynamic cracking strain, ε_{td}. Thus for cracking

$$\varepsilon_1 \geq \varepsilon_{td} \quad \text{and/or} \quad \varepsilon_2 \geq \varepsilon_{td} \tag{3}$$

where subscripts 1 and 2 relate to the principal directions in the plane of a structure. Thereafter, the behaviour of concrete is no longer isotropic. It becomes orthotropic and the local material axes coincide with the principal strain directions. Following the fixed crack approach [1], the crack directions are assumed to be fixed in the directions of the primary cracks, irrespective of the possible rotation of stresses and strains. Under further loading, a secondary crack may occur at a sampling point that was originally cracked in one direction if the strain parallel to the existing crack, ε^*_t, satisfies the cracking condition

$$\varepsilon^*_t \geq \varepsilon_{td} \tag{4}$$

Thus for plane problems, two sets of cracks are allowed at each sampling point in mutually orthogonal directions.

In axisymmetry, the hoop stresses and strains are always principal quantities, and therefore cracks lie in radial planes if the hoop strain satisfies the cracking condition

$$\varepsilon_\theta \geq \varepsilon_{td} \tag{5}$$

The other type of cracks, called the circumferential cracks, can occur in the radial-axial (r-z) plane which are similar to those in planar conditions. Thus, taking into account that this third direction, θ, is always a principal direction and is out of plane, the cracking prediction follows the same procedure of plane problems for circumferential cracks.

CRACK PROPAGATION PROCESS

Tension softening rule

In order to account for the post-cracking residual tensile strength of concrete in the present computational model, it is assumed that the loss of tensile strength occurs gradually after cracking. An important consideration in the selection of crack propagation criterion for the smeared cracking concept is the objectivity with respect to mesh elements size. The element size effect is treated here by an approach proposed by Nilsson et al [8], in which the abrupt stress reduction is replaced with a gradual stress reduction to follow a fracture energy based non-linear softening curve as shown in Figure 1. The fracture energy release rate, G_f, defined as a material property is used with local strain softening rule and a characteristic length, L_c, for the subjective treatment of the post-cracking behaviour. As a result the fracture energy concept leads to a non-local format of the equivalent softening relation.

The stress, σ, across an opening crack is assumed to be a function of crack width, w, [8] such that

$$G_f = \int_0^{\infty} \sigma\ (w)\ dw \tag{6}$$

It was assumed that inside a control volume, v, containing a crack with a surface area A, all inelastic deformations take place in the crack, so the rest of the control volume remains elastic. The rate of energy dissipation in the crack is

$$\dot{U}_A = \int_A \sigma\ \dot{w}\ d\ A \tag{7}$$

In order to replace the discontinuous crack with an equivalent smeared crack, it is assumed that the control volume is subjected to the same state of stress as the crack, but strained by equivalent strain, ε_c. The rate of energy dissipation in this continuous volume is

$$\dot{U}_v = \int_v \sigma\ \dot{\varepsilon}_c\ dv \tag{8}$$

The relationship between the crack width and the fictitious crack strain is obtained by equating (7) to (8), which leads to

$$\frac{dw}{d\varepsilon_c} = \frac{V}{A} \tag{9}$$

Taking the characteristic length of crack as the ratio between the control volume and the crack surface, equation (9) can be re-written as

$$\frac{dw}{d\varepsilon_c} = L_c \tag{10}$$

Using equation (10), the fracture energy in equation (6) is defined as

$$G_f = L_c \int_0^{\varepsilon} \sigma\ d\ \varepsilon_c \tag{11}$$

Based on various experimental evidence, an exponential function is used to simulate strain softening branch [9, 10] as

$$\sigma = \sigma_{ts}\ e^{-\frac{\varepsilon - \varepsilon_{ts}}{\delta}} \tag{12}$$

where σ_{ts} is the static tensile strength of concrete, ε_{ts} is the cracking strain, δ is the tension softening parameter and ε is the normal tensile strain in the cracked zone. From Figure 1, the stress in the linear elastic part can be defined as

$$\sigma = \varepsilon\ \ \sigma_{ts} / \varepsilon_{ts} \tag{13}$$

Substituting (12) and (13) into (11) and upon mathematical manipulation, the tension softening parameter is found [10] as

$$\delta = \left(G_f - \frac{1}{2}\sigma_{ts}\,\varepsilon_{ts}\,L_c\right) / \sigma_{ts}\,L_c \qquad (14)$$

In the context of finite element computations, the control volume for a crack is the volume associated with a sampling point in a given element. The characteristic crack length is computed here for each sampling point [10] as

$$L_c = (dv)^{1/3} \qquad (15)$$

where dv denotes the volume of concrete represented by the sampling point inside each element.

Shear transfer across the cracks

At the onset of cracking, the ability of concrete to transfer shear stresses across the crack is greatly reduced. However, phenomenon such as aggregate interlock and dowel action due to reinforcing bars must be taken into consideration. Both mechanisms are controlled by the width of the crack, the shear transfer capacity being reduced as the width increases. To account for the shear capacity of cracked concrete in the smeared crack approach, a simplified procedure is generally employed by assigning to the uncracked shear modulus, G, a reduced value, G_c, defined as

$$G_c = \beta_s\,G \qquad (16)$$

where β_s is a reduction factor in the range of zero to one. In the present work, the reduction factor is related by a monotonically decreasing function, to the tensile strain normal to the crack plane, a smeared measure of the crack width by the expression

$$\beta_s = 1 - \left(\frac{\varepsilon_n^*}{\varepsilon_m}\right)^{\mu}, \quad \beta_s = 0 \text{ if } \varepsilon_n^* \geq \varepsilon_m \qquad (17)$$

where ε_m is the maximum limiting tensile strain and μ is a parameter in the range 0.3 to 1.0 [11]. The value of the maximum limiting strain is often considered in a range of 0.004 - 0.005.

SOME OTHER FEATURES OF COMPUTATIONAL CRACKING STRATEGY

Closing and opening of existing cracks

The redistribution of stresses due to cracking in other sampling points, or further loading or unloading may force some of the previously open cracks to close partially or fully. The current strain normal to the crack direction is used [1] to assess the state of the cracks in already cracked concrete. Thus, a crack is assumed to be fully closed if the strain becomes negative, as

$$\varepsilon_n^* \leq 0 \quad \text{and/or} \quad \varepsilon_t^* \leq 0 \qquad (18)$$

and/or additionally for axisymmetric problems

$$\varepsilon_\theta \leq 0 \qquad (19)$$

The compressive stresses can then again be transmitted across the cracks. If the strain normal to the crack direction decreases, but is still positive, partial closing of the crack is assumed. This situation may occur when the current strain, ε^*_n, is smaller than the reference strain, ε_{ref}, recorded as the maximum tensile strain reached across the crack under consideration at the previous time steps. In this case, the stress normal to the crack is computed from

$$\sigma_n = \frac{\sigma_{ref}}{\varepsilon_{ref}}\, \varepsilon \tag{20}$$

in which σ_{ref} is the interpolated stress corresponding to the strain ε_{ref}. This secant unloading path is shown in Figure 1. Re-opening of fully closed crack is also monitored by the tensile strain normal to the crack direction, i.e.

$$\varepsilon^*_n > 0 \quad \text{and/or} \quad \varepsilon^*_t > 0 \tag{21}$$

and/or for axisymmetric problems

$$\varepsilon_\theta > 0 \tag{22}$$

In this case, the crack follows the same secant path until ε_{ref} is exceeded and then the stress is interpolated from the strain softening governing equation (12). It should be mentioned that from the moment of first cracking, for each integration point, a record of the crack angle, the crack state, and the cracking reference values (ε_{ref}, σ_{ref}) are kept and constantly updated at each time step.

Control of stress locking phenomenon

Occasionally in the vicinity of fracture localization zones the principal stress plots exhibit stress locking phenomenon when the stress should actually drop to zero. This leads to overstiff response. The stress-locking is a fundamental consequence of finite element displacement continuity in smeared softening approaches. If a certain element contains an inclined crack at sampling point, GP1, the strain imposed by this crack implies adjacent integration points, e.g. GP2, to be strained as well. Hence, while the stress at GP1 correctly softens, the stress at GP2, which is still in the elastic regime, increases. Then, there are two possibilities [12]: the strain at GP2 will either exceed the limiting cracking strain and start softening, or it will not. The former alternative discourages localizations (spurious cracking). but the latter suffers from spurious stiffening. Stress locking does not disappear on mesh refinement, since refinement does not remove the fundamental assumption of displacement compatibility. As a result, the solution of the problem must be primarily sought in improvements of the finite element approximations. It has been suggested [12] that a possibility is to interactively align the elements with the lines of the fracture, or to use some isotropic softening law, whereby not only the stiffness normal to the crack but also the stiffness parallel to the crack deteriorates. However, with the former method the featuring advantage of smeared cracking as to maintaining the original topology is lost. The latter solution is not realistic since in reality the stiffness of concrete parallel to the cracks does not disappear. In the present work, the use of Guassian quadrature technique with the reduced 2x2 order for the numerical integration of the stress fields proved [1] to be a practical and successful means of handling elastic and inelastic domains within the same 8-node isoparametric elements adopted here for geometry discretization. It gives a partial release of the continuity requirements imposed by shape functions of elements.

COMPUTER IMPLEMENTATIONS AND NUMERICAL APPLICATIONS

Crack monitoring algorithm

A new crack monitoring algorithm is developed in [1] which follows the subsequent cracking of elements. The outlines of the algorithm are given in Table 1. This algorithm is part of a versatile and comprehensive computer program, FEABRS, which has been developed [1] for the finite element linear and non-linear dynamic analysis of plane and axisymmetric reinforced concrete structures under blast and acceleration history excitations. Explicit central difference scheme has been employed for the time discretization of the governing dynamic equilibrium equations. The crack monitoring algorithm is activated after strain evaluation in each time step and for each cracked integration point.

Beam under impulsive concentrated load

A simply supported reinforced concrete beam is subjected to two symmetrically applied concentrated loads which are applied as step loads with a zero rise time. The problem has been solved in [10, 13]. The beam is reinforced in the lower position by 1290 mm^2 steel area. The material properties of concrete and steel are listed in [1]. Using symmetry conditions, only one half of the beam is modelled using the 8-noded isoparametric elements with axial bars to simulate the reinforcing steel. No viscous damping has been considered. The time step size was chosen as 0.000002 sec.

The performance of the rate independent modelling is compared with the rate dependent response of the beam in Figure 2 which shows the variation of the mid-span deflection with time for the non-linear analyses together with the elastic response. Generally, the results of the linear and non-linear rate dependent analyses found to be in good agreement with those reported elsewhere [10, 13]. The following points can be drawn from the Figure:

1. Compared with the linear analysis, the much longer response (almost 50% higher) of non-linear cases is a result of the softening behaviour of the structure with the progression of cracks in the structure.
2. For the non-linear analyses, both the peak displacement and the time corresponding to the peak deflection vary with the rate sensitivity consideration. Ignoring the strain rate effect results in an over-estimate by almost 12% of the maximum deflection obtained for the rate dependent case, and by approximately 64% compared with the elastic case. An elongation of the natural period can also be noticed. Based on the formulation of the proposed concrete constitutive modelling to simulate the experimentally observed phenomena under dynamic loading, two differences exist between the rate dependent and rate independent models that may account for differences observed in the deflection history. First, the inclusion of strain rate effect results in stress-strain diagrams which indicate higher strength and greater energy absorption before concrete fractures. Second, the adopted rate dependent crack initiation criterion introduces numerically crack arresting mechanisms associated with the higher limiting cracking strain.

To study the sensitivity of the beam response to variations in the concrete fracture energy, the non-linear analysis is performed with G_f varied between 105.1 and 150.6 Nm/m^2. It can be observed from Figure 3 that the consequence of reducing this material property is an amplification of the effect of concrete cracking which results in an increase in deflection and an elongation of the period. However, the increase in the deflection is not proportional to the change in the fracture energy. As the fracture energy is increased by 50%, the maximum deflection decreases by only 10%.

The beam topology is traced in Figure 4 which shows the deformed shape of beam and how the cracks spread in concrete at different time stations. The deformed beam elevation is represented by considering the horizontal and vertical displacements magnified 20 times. Following the cracking history, the beam behaviour is linear elastic in the initial stages. At time, t = 0.00298 sec, cracking commences near the bottom surface of the beam. With increasing deflection, the cracks develop on the underside of the central part of the beam. The cracking patterns indicate that flexural deformations produce the cracks which are all perpendicular to the tension initiation. As the cracks propagate further into the beam, the compressive strain combines with the shear strain and alters the inclination of the principal tensile strain so that the crack inclines towards the horizontal. The significant increase in displacements with time also reveals the fact that the main factor contributing to the non-linear behaviour is the cracking of concrete.

Circular slab under uniformly distributed dynamic force

The slab is subjected to a uniformly distributed load of intensity 137.3 kN/m^2. This problem has been previously analyzed by several investigators [10, 14]. The plate has a radius of 10 m and a thickness of 1 m. The load is applied with a rise time equal to half of the elastic fundamental period (T = 0.06 sec). The percentage of reinforcement placed near the upper and lower surfaces in the radial and tangential directions is 1%. The material properties employed here are given in [1]. Following the axisymmetric conditions, only a quadrant of the plate is modelled in the r-z plane using 24 isoparametric elements. The reinforcement was simulated as axisymmetric membrane with Poisson's ratio $\nu = 0.0$, which in the adopted model implies four steel layers, each of 1.0 cm equivalent thickness. The analysis was performed with a time step of 0.000005 sec and with no damping.

The numerical results of the time history of mid-span deflection in Figure 5 are presented for linear analysis as well as non-linear response for two different levels of concrete cracking strains. The linear elastic curve and the curve obtained for cracking strain 0.00018 are in good agreement with the results reported in [10,14]. The general trends observed in the previous example are confirmed. Non-linear effects amplify displacements and elongate the period of vibration. The significant variation in responses for different cracking limits indicates the capability of the proposed computational strategy of the cracking process in predicting the behaviour of concrete structures where tensile cracking is the most sensitive and dissipative mechanism.

At different time intervals, the deformed profiles of one half of the structure and the spread of the crack zones through the thickness in the radial and hoop directions are shown in Figure 6 where the deformations are enlarged 20 times. In the initial stages (time = 0.024 sec), the slab behaviour is linear elastic. At time, t = 0.032 sec, the non-linear analysis leads to cracking in the tangential direction in the vicinity of the built-in section at the top of the slab. The first circumferential cracks are predicted at the bottom of the centre of the slab at time, t = 0.0366 sec. With increasing time, the radial cracks spread to the top of the slab at mid-span and to the bottom of the slab at the fixed edge. On the other hand, the hoop cracks extend from the lower face towards the centre at the top of the slab. It can also be seen from the Figure that the increase in deformations is mainly determined by the increasing displacements in the cracked zones.

CONCLUSION

The main conclusions that can be drawn from the present study are:

1. The proposed computational strategy for modelling the cracking process in smeared fashion is a suitable and powerful tool for the non-linear dynamic analysis of planar and axisymmetric reinforced concrete structures under impulsive and transient loading conditions. Such methodology has been shown to be suited for representing overall behaviour as well as for simulating different crack patterns.

2. For all types of concrete structures, tensile cracking process plays a dominant role in the non-linear response. The numerical results indicate that the structural response is sensitive to the crack initiation parameters (cracking strain and strain rate dependency). Decreasing the limiting cracking strain or ignoring strain rate effect results in higher deflections and longer periods. The proposed rate dependent crack initiation criterion numerically introduces a crack arresting mechanism associated with the higher limiting cracking strain.

3. The computed structural response clearly depends on the post-peak tensile behaviour of concrete through concrete fracture energy G_f. The difference in deflection response for lower values of G_f seems to be a direct consequence of the decrease of the effective load-carrying area of concrete in tension. As the slope of the descending branch of the tensile stress-strain curves decreases for higher values of G_f, the cracks propagate more slowly and the load capacity of the structural member increases. The insensitivity of the response to small changes in fracture energy may be due to the fact that increasing the fracture energy primarily stiffens and does not strengthen the structural element.

4. The plane 8-noded isoparametric elements employed for geometry discretization proved to be a successful and computationally economical means of handling elastic and inelastic spatial domains. Moreover, the use of Guassian quadrature technique with the reduced 2×2 order for the numerical integration of the element stress fields, gives reasonable cracking patterns as well as a control of the stress locking phenomenon as a result of the partial release of the continuity requirements imposed by the shape functions.

5. The crack monitoring algorithm developed as a part of the computer program FEABRS offers good possibilities for non-linear dynamic analysis of concrete structures where strain rate dependency effects, tensile strain-softening, crack shear and partial or full closing and opening of cracks are taken into account. The progressive cracking in

concrete can be traced where single mode or mixed mode fracture analysis can be performed.

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TABLE 1 The crack monitoring algorithm in FEABRS

A - Check whether the sampling point (GP) is previously cracked

B - if not previously cracked (crack has just initiated):

1. - Compute and store crack angle
2. - Set appropriate flags for material state (MSTAT)
3. - Stress evaluation for cracked state, go to D

C - if previously cracked:

1. - Compute strains in local coordinate system.
2. - Check whether each of the old cracks is opened by checking the dilation across each existing crack.
3. - For plane problems, if GP (Guass Point) is previously cracked in one direction, check whether a new crack has just opened orthogonal to the direction of first one by checking the strain parallel to the first crack.
4. - For axisymmetric problems, if GP is previously cracked in one direction or two directions check whether new cracks (two for the first case and one for the second) just initiated from the following three possibilities of cracks:

i. radial crack by checking the strain in the hoop direction.

ii. main circumferential crack by checking the strain in the radial direction.

iii. secondary circumferential crack orthogonal to the main one by checking the strain parallel to the main circumferential crack.

5. - Store angle of each new crack
6. - Update appropriate flags for MSTAT
7. - Stress evaluation of cracked state

D - stress evaluation procedure for cracked state:

1. - Evaluate stresses in crack directions using suitable constitutive relationship for cracked concrete, [1].
2. - Update the normal stresses across the cracks using tension softening rule.
3. - Update shear stresses across the cracks using shear transfer model.
4. - Update the crack reference values.
5. - Transform stresses back to global directions.

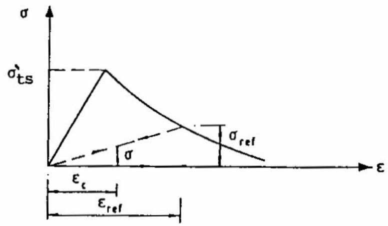

Figure 1 Strain softening curve with secant unloading and reloading

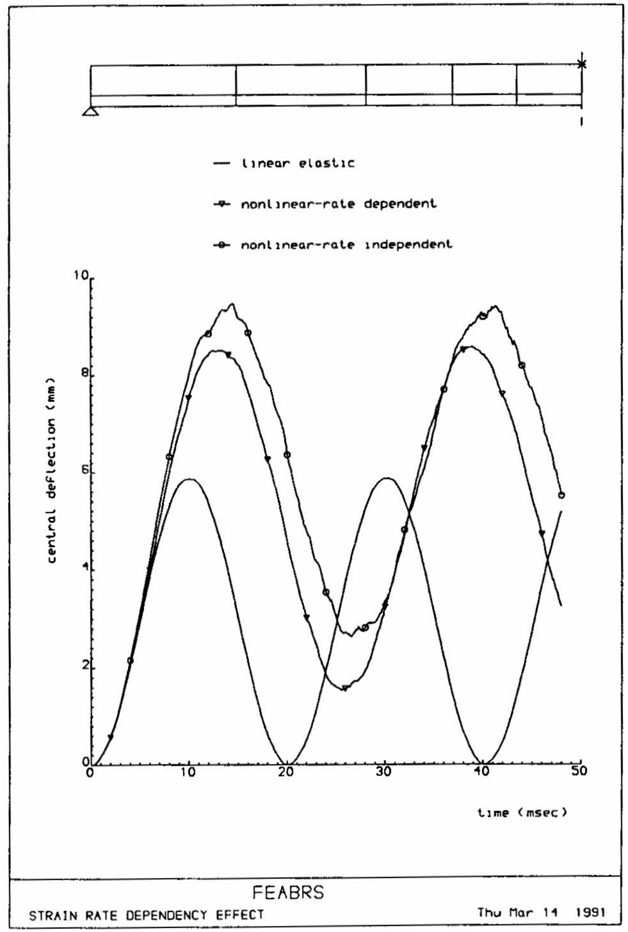

Figure 2 The effect of strain rate on the nonlinear response of Bathe's beam

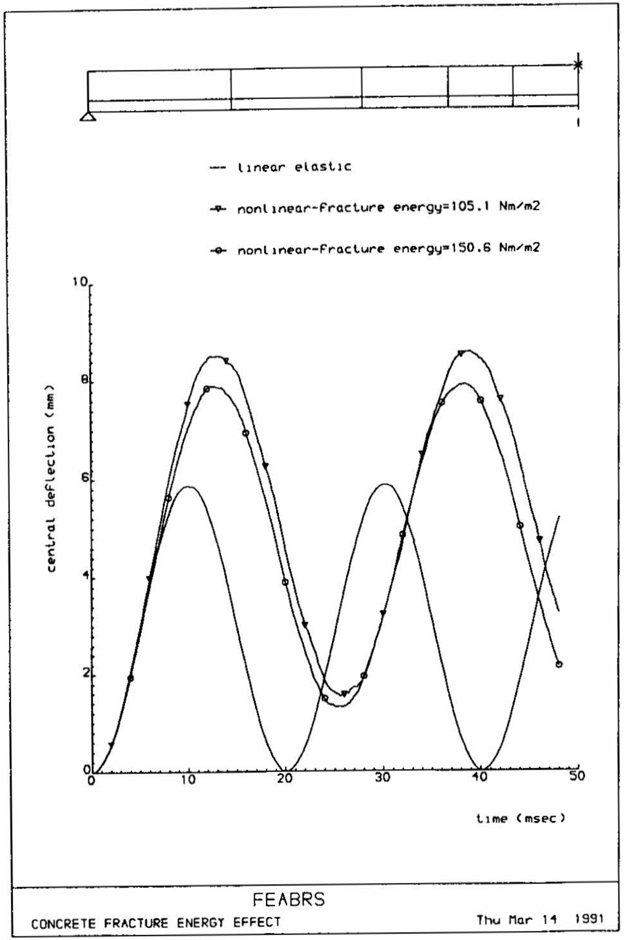

Figure 3 Influence of concrete fracture energy on nonlinear response of Bathe's beam

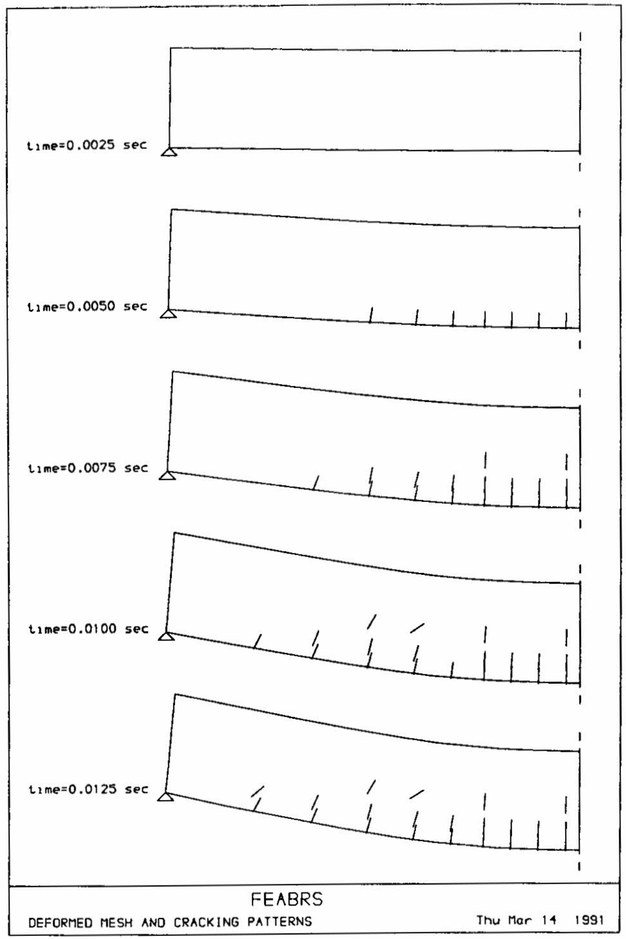

Figure 4 Deformation and cracking history of Bathe's beam

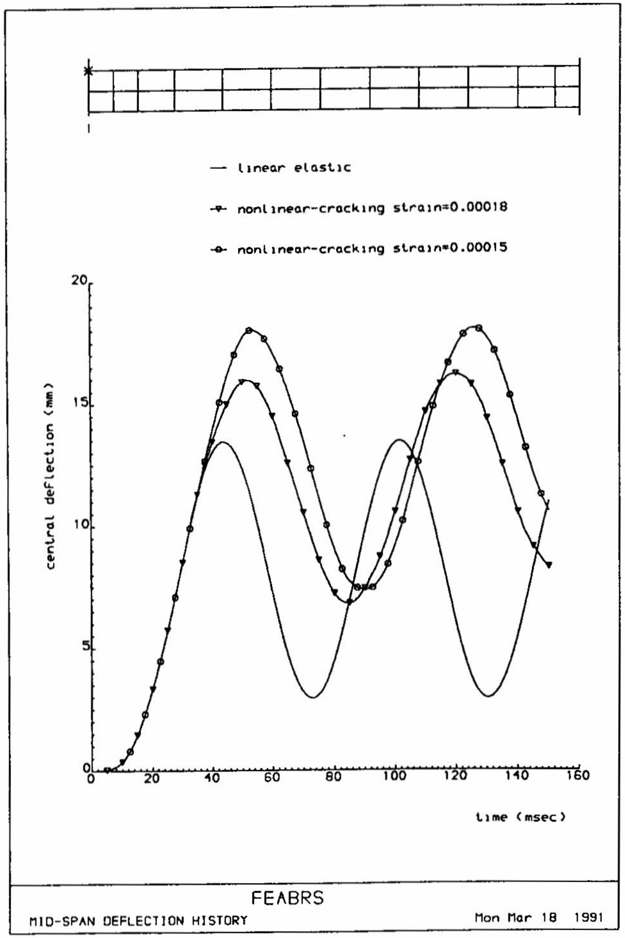

Figure 5 Nonlinear response of circular plate

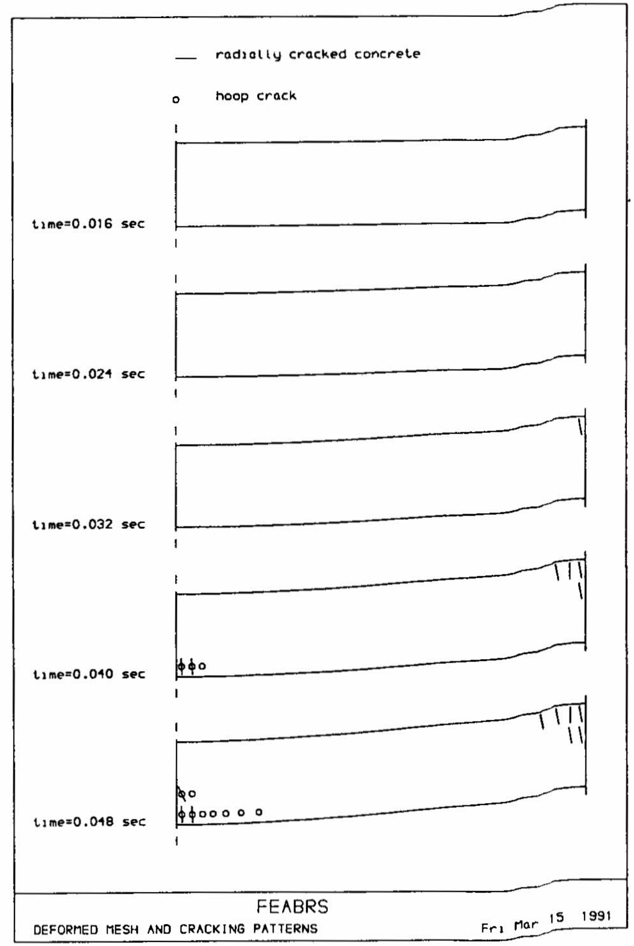

Figure 6 Deformation and cracking history for circular slab